Semiparametric modeling of age-specific variations in income related health inequalities

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Abstract

A Gini-type concentration index is combined with semiparametric estimation techniques to derive a varying inequality index that works without a priori sample stratification. The new approach is used to investigate the question how income inequalities and the income-related gradients in the distribution of health vary across age groups. With health data from the 2005 survey of the German microcensus it is demonstrated that significant inequalities to the detriment of the deprived evolve in early mid-life and reach their maximum around the age for retirement. Some leveling is found for the elderly.

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1 Introduction

The existence of socioeconomic gradients in the distribution of health to the detriment of the deprived is firmly established among health economists (Balía and Jones, 2008; Erreygers, 2009; Humphries and van Doorslaer, 2000; van Doorslaer et al., 1997; van Doorslaer and Koolman, 2004; van Doorslaer et al., 2004; Jones and López Nicolás, 2006; Wagstaff et al., 1991; Wagstaff and van Doorslaer, 2000; Wagstaff et al., 2003). Little is known, however, about the mechanisms through which different socioeconomic factors affect health status and its distribution over the life course (van Kippersluis et al., 2009, 2010). Adding the life
course perspective supports, for instance, the notion that labor force participation contributes substantially to the socioeconomic gradient in health in the U.S. (Case and Deaton, 2005), Great Britain (Banks et al., 2007) and the Netherlands (van Kippersluis et al., 2010).

To measure variations in health inequalities across age groups, van Kippersluis et al. (2009) define age cohorts and compute batteries of concentration indices for eleven European countries. This paper derives a varying inequality index for dichotomous health variables based on the concentration index that does not require a priori sample stratification. One may consider estimating a nonparametric smoother through age-specific concentration indices as an alternative, however, this approach would reduce the reliability of the results considerably. The number of observations in arbitrarily predefined age groups may be rather small resulting in high uncertainty particularly among the oldest age groups. Further, the estimates for ages close to the upper (lower) cohort limits would only be subject to the younger (older) individuals within the same cohort and hence likely be biased. Smoothing over such results in a second step would then add its own uncertainty and lead to fairly imprecise results. Based on the varying coefficient model (Hastie and Tibshirani, 1993; Li et al., 2002), we propose a semiparametric extension of the convenient regression approach (Kakwani et al., 1997). Using kernel smoothing techniques and a locally chosen bandwidth allows us to estimate the functional relationship between the concentration index and age. We adjust our varying index adapting Wagstaff’s (2005) correction formula for binary variables with local estimates of the mean.
2 Methods

2.1 The concentration index

The concentration index $C$ stems from the concentration curve, where the cumulative share of some health variable $y$ is plotted against the cumulative share of the population ranked by socioeconomic position. It measures twice the area between the concentration curve and the line of equality and is bound in the $(-1; 1)$ interval. $C$ becomes positive (negative) if the variable of interest concentrates among the rich (poor) and is zero if no income-related inequality is observed (Wagstaff et al., 1991).

Using the covariance approach (Lerman and Yitzhaki, 1989), Kakwani et al. (1997) present the regression formula for $C$,

$$\frac{2\sigma^2_y}{\mu_y} = \beta_0 + \beta_1 r + \epsilon,$$  \hspace{1cm} (1)

where $\mu_y$ is the mean of $y$ and $r$ is the fractional rank with variance $\sigma^2$. Equation (1) can be estimated using linear regression models such that $C = \beta_1$.

2.2 Varying coefficient models

In the framework of varying coefficient models, Li et al. (2002) propose a semi-parametric smooth coefficient model based on locally weighted least squares regression. With $X$ denoting the regressor matrix and $y$ the dependent variable, the elements of the coefficient vector $(\beta_0, \ldots, \beta_Q)$ are modeled as smooth functions of
another regressor $z$, which varies in some $Z \subset \mathbb{R}$:

$$y = \beta_0(z) + \sum_{q=1}^{Q} \beta_q(z)x_q + \varepsilon. \quad (2)$$

This model can be estimated using nonparametric smoothing techniques (see Li et al., 2002; Hastie and Tibshirani, 1993) from

$$\hat{\beta}(z) = (E(X'X \mid z))^{-1} E(X'y \mid z), \quad (3)$$

where $X = (1 \ x_1 \ldots x_Q)$ and $z \in Z$. Li et al. (2002) have shown that, for an increasing number of observations $n$, the estimator $\hat{\beta}(z)$ obtained from (3) asymptotically follows a normal distribution, i.e. $\sqrt{m/n} \left( \hat{\beta}(z) - \beta(z) \right) \sim N(0, \Omega(z))$; see appendix for the estimation of the covariance matrix.

### 2.3 A semiparametric inequality index

Combining the weighted regression approach from equation (1) with the varying coefficient model (2), our proposal for a semiparametric convenient regression formula is

$$2 \frac{\sigma^2_y(z)}{\mu_y(z)} y = \beta_0(z) + \beta_1(z) r(z) + \varepsilon, \quad (4)$$

with $C(z) = \beta_1(z), z \in Z$. Note that the concentration index is a bivariate extension of the Gini index; if $y$ is the social status variable, equation (4) works as a semiparametric Gini index. The local mean $\mu_y(z)$ can be estimated nonparametrically. The weighted fractional rank $r(z)$ has to be written as a function of $z$ because the condition that its mean and variance have to be .5 and $1/12$, respectively, must hold (Lerman and Yitzhaki, 1989). This can only be fulfilled if the weighted fractional
rank is computed using only those individuals included in the local regression and incorporating the kernel weights \( k_{h_z}(u_i) \):

\[
  r_i(z) = \sum_{j=1}^{i} w_j(z) k_{h_z}(u_j) - \frac{w_i(z) k_{h_z}(u_i)}{2}.
\]

The vector of sample weights \( w(z) \) must be rescaled such that \( \sum_{i=1}^{n} w_i(z) k_{h_z}(u_i) = 1 \) for each \( z \in Z \). Note that the mean \( \mu_r(z) = .5 \) and variance \( \sigma^2_r(z) = 1/12 \) of \( r(z) \) are then sample independent and do not vary with \( z \).

For binary variables, the bounds of \( C \) depend inversely on the variable’s mean, \(|C| \leq 1 - \mu_y\) (see Wagstaff, 2005, 2011; Erreygers, 2009). For an intuitive explanation, first assume a constant equal to 1. With no difference between individuals, concentration among rich or poor is impossible; \( C \) equals zero. Now consider, say, 10 percent ones and 90 percent zeros. Ordering the variable by itself, one would obtain a Gini index of .9; the largest possible concentration (see Wagstaff, 2011, for a graphical illustration).

There is an ongoing discussion on possible correction methods for concentration indices of limited variables (Erreygers, 2009; Wagstaff, 2005, 2011) with a dissent between the authors on how a corrected index should react to changes of the mean. Considering the above example, Erreygers (2009) would argue that an increase from 10 to 20 percent implies a decrease in inequality as now the second richest (poorest) decile is also affected. The argument in Wagstaff (2005, 2011) is that, as still only the richest (poorest) are affected, the new situation still corresponds with the maximum possible inequality. We consider reactions to pure prevalence changes as undesirable when comparing inequalities between age groups and sexes. We thus propose adapting the formula in Wagstaff (2005,
2011) as a pointwise correction of the semiparametric concentration index using
the local mean $\mu_y(z)$ of $y$:

$$W(z) = \frac{C(z)}{1 - \mu_y(z)}.$$  \hspace{1cm} (6)

Note that our approach may also be adapted to other correction methods for rank-
dependent inequality indices such as mentioned e.g. by Erreygers (2009).

2.4 Estimation

Applying a consistent Nadaraya-Watson estimator, we account for sample weights
and use

$$\hat{\beta}(z) = \left[ \sum_{i=1}^{n} k_{h_z} (u_i) w_i(z) X_i' X_i \right]^{-1} \left[ \sum_{i=1}^{n} k_{h_z} (u_i) w_i(z) X_i' \tilde{y}_i \right]$$  \hspace{1cm} (7)

to obtain $\hat{\beta}_1(z) = \hat{\beta}(z), z \in Z$. Note that $\tilde{y}_i = (\hat{\sigma}^2_i(z)/\mu_y(z)) y_i$ and $X_i = (1 \cdot r_i(z))$
depend on $z$ as the local mean and the local fractional rank from equation (5) are
involved (for simplicity, we write $X_i$ in place of $X_i(z)$ here). The kernel weights
are $k_{hx} (u_i) = K_{hx} (u_i) \left[ \sum_{j=1}^{n} K_{hx} (u_j) \right]^{-1}$ with $u_i = z_i - z$. We use the quartic kernel
$K(u_i) = (15/16) (1 - u_i^2)^2 I_{|u_i|<1}$ with $\|K_2^2\| = \int_{-\infty}^{\infty} K^2(u) du = 5/7$, where $I_A$ is an
indicator function of restriction $A$. The bandwidth $h_z$ is included such that $K_{hx} (\cdot) =
(h_z)^{-1} K (\cdot/h_z)$. The quartic kernel assigns higher weights to observations closer to
$z$ (smaller $u_i$), lower weights for observations further away from $z$ (larger $u_i$) and
zero weights if an observation is outside the bandwidth. Although our estimator
is asymptotically unbiased (Li et al., 2002), nonparametric regression in finite
samples always suffers from a tradeoff between bias and variability: decreasing
bandwidth parameters yield smaller biases but also increase uncertainty (Bilger,
2008). This problem is addressed here by choosing the bandwidth inversely to the

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data density as $h_z = 1.06 \hat{\sigma}_z n^{-\frac{2}{d}} \hat{f}_z^{-3}$, where $\hat{f}_z$ is the estimated kernel density at a particular value of $z$ and $\hat{\sigma}_z$ is the standard deviation of $z$ obtained from the sample. Fan and Gijbels (1992) have shown that adaptive local smoothers generally yield good results and, in addition, avoid the well-known boundary effect.

Kakwani et al. (1997), O’Donnell et al. (2008) and Wildman (2003) argue that it is not sufficient to estimate the standard error of $\hat{\beta}_1(z)$ from equation (1) and propose approximating the standard error of $C, \sigma_C$, using the $\delta$-method. Further, using the covariance matrix from simple OLS regression may not be wholly accurate as the error term may be autocorrelated and heteroscedastic (Kakwani et al., 1997). Wildman (2003) proposes using the order of the rank variable in place of time to compute heteroscedasticity and autocorrelation consistent Newey-West covariance matrices. We apply the $\delta$-method to equation (6) and compute locally estimated heteroscedasticity and autocorrelation consistent covariance matrices to obtain the local standard error $\sigma_W(z)$ of $W(z)$ (see appendix for details).

3 Data and variables

We use the scientific use file (SUF) of the German microcensus (Mikrozensus) for the empirical illustration. The SUF comprises a randomly drawn subsample of approximately 70% (477,239 observations) of the German microcensus and is available from the Research Data Centers of the Federal Statistical Office and the Statistical Offices of the Federal States (see Lechert and Schimpi-Neimanns, 2007, for a technical report). Removing 92,458 individuals from the sample owing to missing information leaves us 384,781 observations (198,877 female and 185,904 male) for the empirical analysis. The inverse probability weights ac-
counting for the regional, age and sex specific composition of the sample were adjusted accordingly.

We use a subjective measure of health and restrict the analysis to having been ill in the preceding four weeks or not, thus generating a binary variable. This measure may include (common) acute diseases such as colds or light flues, however, we postulate here that these affect all socioeconomic groups. Assuming that only those who actually felt affected would consider themselves as ill, this measure of health has the advantage that it only includes diseases if they were relevant to the respective individual.

Net equivalent household income (based on the modified OECD equivalent scale and not restricted to a particular source of income) is used to assess an individual’s relative socioeconomic position (see e.g. van Doorslaer et al., 2004; van Kippersluis et al., 2009). One may consider this as unsuitable for some countries particularly after retirement; however, Germany is an exemption because its welfare policies can be seen as rather status preserving (Brockmann et al., 2009). Approximately 90 percent of the German population are covered by the public pension system where benefits after retirement depend on compulsory contributions subtracted from the gross income (for a description of the German public pension system, see Boersch-Supan and Wilke, 2004). Considerable changes of the relative socioeconomic position within one’s age group after retirement seem therefore unlikely.
Figure 1: Empirical density of age (left) and smoothed age specific prevalence of sickness within the preceding four weeks (right) for males (solid line) and females (dashed line)

4 Results

The left graph in figure 1 describes the kernel density estimate \( f_z \) of the nonparametric smoothing regressor \( z \) (age), corresponding with the population pyramid for Germany. The graphs for the male and female sample imply that the largest bandwidth parameter was used for subjects older than 80 in both samples. The right graph in figure 1 presents the smoothed age specific prevalence of illness within the preceding four weeks.

The upper and lower left graphs in figure 2 present the age-specific means of the net equivalent household incomes for males and females. Considering that income is assigned equally to each household member, the graph suggests that households with children have, on average, the lowest income. The bump around age 40 in both graphs stems from the higher average number of dependent children which increases the equivalence weights in the corresponding households. The age-specific income inequalities vary around their homogeneous counterparts (female: 0.282, male: 0.2915).
Figure 2: Age-specific mean (left) and Gini indices (right) with 95% confidence intervals (dotted lines) of net equivalent household income for males (top) and females (bottom).

The overall-sample results suggest a considerable concentration of illness among the poor. The homogeneous concentration indices are $-0.0606$ ($\sigma_c = 0.0026$) for the female and $-0.0653$ ($\sigma_c = 0.0028$) for the male sample. The homogeneous Wagstaff indices are $-0.0696$ ($\sigma_w = 0.0074$) for females and $-0.0738$ ($\sigma_w = 0.0079$) for males.

The varying Wagstaff indices in figure 3 vary around the homogeneous estimates. Illness is significantly concentrated among males in lower income households in the age groups between 19 and 26 as well as between 33 and 77. The index for females shifts from a concentration among the better-off towards a concen-
Figure 3: Age-specific inequality (smooth solid lines) with 95% confidence intervals (dotted lines) for males (top) and females (bottom); Wagstaff index computed for five year interval age groups (solid step function)

...tration among the worse-off in late childhood. Significant concentration among females in lower income households is found for age groups 17-25 and 39-74. Considerable sex-specific differences in the curve shapes are only found for the 20 to 40 years old. While there is some flattening for the male sample, inequality is comparably low among females in this age group.

The results from the index computed for five year age groups are similar to those from the varying index where data are dense. However, the five year interval index exhibits several leaps and a considerably higher variability. Comparison of the two graphs demonstrates the above mentioned bias close to the group limits.
We also computed the inequality index and the five year interval Wagstaff indices for 5%, 10%, 25% and 50% subsamples with similar results. The confidence bands of the varying inequality index widen and the variability of the Wagstaff indices computed for five year intervals increases with decreasing $n$. The graphical comparisons suggest that the index performs well for samples with more than 30,000 observations (the results are available from the authors on request).

5 Discussion

In this article, we combine the notion of concentration indices (Erreygers, 2009; Kakwani et al., 1997; van Kippersluis et al., 2009; Wagstaff et al., 1991; Wagstaff, 2005) with semiparametric regression techniques (Hastie and Tibshirani, 1993; Li et al., 2002) to a semiparametric inequality index with some convenient properties. Using the varying bandwidth inverse to the local density, the index adapts itself to the data without a priori stratification into age or income groups. This method allows an age-specific computation of the inequality index with a sufficiently large number of observations guaranteed even where observations are scarce. The local correction based on Wagstaff’s (2005) formula allows comparisons of the extent of inequalities throughout the support of the smoothing regressor.

Using German microcensus data, we demonstrate the advantage of the semi-parametric index illustrating that direction and extent of health inequalities vary considerably across age groups in Germany. According to Dupre (2007), leveling in old age may generally be an artificial effect owing to mortality selection as both decline of health and increase of mortality rates are faster among the worse-off. However, Beckett (2000) has shown that this needs not necessarily be true for self-
reported health. Note that mortality rates in 2005 did not exceed 2 percent before the age of 68 (74) and 5 percent before the age of 77 (81) in the male (female) sample (see Human Mortality Database, 2011). The results for those older than 80 should be treated with caution, though, as mortality may play a considerable role in these age groups.

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**A Variance estimation**

Kakwani et al. (1997), O’Donnell et al. (2008) and Wildman (2003) have shown that one may estimate $\hat{\beta}_0^*(z)$ and $\hat{\beta}_1^*(z)$ from $y = \hat{\beta}_0^*(z) + \hat{\beta}_1^*(z) r(z) + e^*$ and consider the concentration index as a nonlinear combination of the two coefficients. The variance can be approximated using the $\delta$ method (Rao, 1965) on $C(z) \approx 2\sigma_r^2(z) [\hat{\beta}_0^*(z) + \mu_r(z) \hat{\beta}_1^*(z)]^{-1} \hat{\beta}_1^*(z)$ for the semiparametric concentration index, the variance of the varying Wagstaff index $W(z)$ can be estimated analogously.

According to Li et al. (2002), the covariance matrix $\Omega(z)$ in the semiparametric varying coefficient model is

$$\Omega(z) = \left[ f_z E (X'X \mid z) \right]^{-1} \Phi(z) \left[ f_z E (X'X \mid z) \right]^{-1}$$  \hspace{1cm} (8)
with $\Phi(z) = f_z E \left( X' X \sigma^2(z) | X, z \right) \| K_2^2 \|$, $\sigma^2(z) = E (\epsilon_i^2 | X, z)$ and $z \in Z$. To estimate a heteroscedasticity and autocorrelation consistent covariance matrix $\hat{\Omega}_{hac}(z)$ accounting for the possibility of heteroscedastic and autocorrelated error terms $\epsilon^*$, $\Phi(z)$ must be adapted accordingly. Following the proposal by White (1980), $\Phi(z)$ is computed as

$$\hat{\Phi}_{hac}(z) = f_z \left( \Psi_0(z) + \sum_{j=1}^{m} \omega_{jm} \Psi_j(z) \right) \| K_2^2 \|. \quad (9)$$

with

$$\Psi_j(z) = \sum_{i=j+1}^{n} k_{h_t}(u_i) w_i(z) \epsilon_i \epsilon_{i-j} \left( x_i x'_i - x_i - x_{i-j} x'_{i-j} \right), \quad (10)$$

$\Psi_0 = \sum_{i=1}^{n} k_{h_t}(u_i) w_i(z) \epsilon_i^2 x'_i x_i$ and $z \in Z$. Bartlett weights $\omega_{jm} = 1 - j/(m+1)$ are applied to assure a positive semi-definite covariance matrix (Newey and West, 1987), $E (X' X | z)$ and the kernel density are computed as above.

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