Asymmetries in Business Cycles and the Role of Oil Prices

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Abstract

We estimate asymmetries in innovations to Solow residuals for eleven OECD countries using Stochastic Frontier Analysis. Likelihood ratio statistics and variance ratios imply that all countries with net energy imports have significant negative asymmetries, while other countries do not. Conditioning Solow residuals on Hamilton’s (2011) net oil price increase variable reduces evidence of negative asymmetries. We construct a simple theoretical model which yields an asymmetric response of measured TFP for oil importers to oil prices. The asymmetric response is due to the effect of the endogenous adjustment of imported energy on value added measured in constant dollars.

Keywords: Solow residuals, stochastic frontier analysis, oil prices, financial crises

JEL Classification: E32, C22, C13

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1 Introduction

There is considerable evidence that business cycles are asymmetric. Recessions tend to be short and deep, while expansions are mild and protracted. Yet Dynamic Stochastic General Equilibrium (DSGE) models attribute a large fraction of the variance of output to symmetric disturbances to total factor productivity (TFP). Are DSGE models missing asymmetric TFP shocks? The first goal of this paper is to determine whether OECD countries have symmetric or asymmetric innovations to TFP, measured by Solow residuals. We reject the null of symmetry for most countries. Our method is atheoretical, with findings of asymmetry completely independent of potential causes. However, there is a notable distinction dividing countries with asymmetries from others. All countries with asymmetries are net energy importers while none of the countries with symmetry are, leading us to consider the role of energy prices in creating the asymmetry.

Our focus on the symmetric properties of TFP is motivated by Barro (2006). In an attempt to explain the equity premium puzzle created by DSGE modeling, he postulates an asymmetric time series process for total productivity. Specifically, he assumes that the log of total factor productivity evolves as a first order autoregressive process with a constant mean and an error with two components. One component is i.i.d normal, capturing symmetric productivity shocks, as in the standard DSGE model. The second component is asymmetric and has a distribution containing only negative values. It captures rare disasters, which Barro defines to include war, natural disasters, financial crises – any event severe enough to create at least a fifteen percent cumulative fall in GDP.

In this paper, we assume that total factor productivity, measured by the Solow residual, has a symmetric i.i.d. component and an asymmetric component with only negative values, as in Barro (2006). In contrast to Barro, we test for the presence of asymmetry in TFP instead of measuring it as large negative GDP outcomes. We compute the innovation in the Solow residual\(^1\) and use Stochastic Frontier Analysis (SFA) to estimate the model. SFA is typically employed in the micro literature to study firm-level productivity in a cross-section. We adapt it to estimate the extent of asymmetry in aggregate time series. We compare the restricted model without asymmetries to the unrestricted model using a likelihood ratio test. Additionally, we estimate the ratio of variances for the two

\(^1\)To compute the innovation, we first difference the logarithm of the Solow residual to generate a stationary time series. Then we regress the log differenced Solow residual on sufficient lagged values of itself to produce a residual without autocorrelation.
components of the innovation to compare the extent of asymmetries across countries. Our sample consists of eleven OECD countries which had quarterly data available back at least to the early 1980’s on GDP, employment, and investment. We reject the null of symmetry for most countries in our sample, generally confirming Barro’s hypothesis on asymmetry in TFP.

Our work testing for asymmetry in TFP is related to other empirical work on business cycle asymmetry. One approach is to measure amplitude and frequency of expansions and recessions. McKay and Reis (2008) confirm that contractions are briefer and more violent when measured by employment but not when measured by GDP. NBER business cycle dating generates recessions with smaller duration and frequency than booms for the US. Another approach postulates Markov switching between expansion and recession regimes. Markov switching models (Hamilton 1989, Raymond and Rich 1997, Clements and Krolzig 2002, Engemann et al. 2011) similarly find that recession regimes are systematically different from expansion regimes. These approaches tend to find asymmetry, and like ours, none require specification of potential determinants of asymmetry as a component of the empirical tests.

After establishing the existence of asymmetries in TFP for many countries, we use the variation in asymmetries across countries to understand the underlying causes of the asymmetry. We find that all countries, which are net importers of energy, have negative asymmetries and that no net-exporting countries do. This suggests that energy-importing countries are structurally different from energy-exporting ones. There is a large empirical literature on output responses to oil prices. Hamilton (1983) demonstrated that all but one post World War II recession in the US were preceded by a large rise in oil prices. Mork (1989) and Hamilton (2011) have argued that business cycles exhibit asymmetric responses to oil price shocks, but Kilian and Vigfusson (2011a,b) fail to reject the null of no asymmetry in a VAR with oil prices and GDP. Kilian (2008) finds negative responses in the US to oil events, measured as international supply disruptions. Engemann et al. (2011) argue that the probability of switching to a recession regime in a Markov switching model depends on the price of oil for the US and several other OECD countries. Hall (1988) explicitly addressed TFP instead of output and found dramatic falls in productivity with oil price increases.

We consider whether conditioning measured TFP, not only on its own lags but also

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2Our focus is on asymmetry in TFP, measured with Solow residuals, not in total output, but the two concepts are closely related since TFP is one component of the production function.
on the lags of particular asymmetric variables, can reduce the asymmetry in the residual. Hamilton’s net oil price increase variable is an asymmetric variable that could explain some of the asymmetry in measured TFP for net energy importers. We determine whether non-linear conditioning on oil prices, using Hamilton’s (2003) net oil price increase variable, can eliminate the negative asymmetries in TFP for energy importing countries. We find that this conditioning reduces negative asymmetries, measured by variance ratios, for all energy-importing countries except Korea, and completely eliminates negative asymmetries for many. However, negative asymmetries do remain for some countries.

Next, we condition on financial crises, following Barro (2006). Conditioning only on financial crises, and not on oil prices, reduces negative asymmetries, but does not eliminate them. When we condition on financial crises and on large oil price increases, all evidence of negative asymmetries vanishes.

Our results demonstrate that conditioning on net oil price increases and financial crises eliminates negative asymmetric innovations in Solow residuals for OECD countries. Does this imply that we should model the TFP innovation as having an exogenous symmetric component and an exogenous asymmetric component, with the asymmetric component being net oil price increases and financial crises? To address this question we use economic theory to study the interaction of prices of imported energy and Solow residuals. We build a simple model in which Solow residuals are determined by a combination of exogenous technology and endogenous imported intermediate inputs. In the model, the endogenous component of the Solow residual responds asymmetrically to a change in energy prices for energy importers. The endogenous and asymmetric response of TFP to changes in energy prices occurs whether energy prices are endogenous or exogenous to the country’s own exogenous technology.

The rest of the paper is organized as follows. In Section 2 we introduce our econometric methodology. Section 3 presents the data and the baseline empirical results. Section 4 presents additional results on asymmetries after conditioning on oil prices and on financial crises. Section 5 develops a theoretical model to understand the determinants of the asymmetries. Section 6 concludes.

2 Econometric Methodology

In this section, we present an econometric model to investigate and test whether Solow residuals have an asymmetric component. Asymmetry is a key issue in the literature on
production efficiency. The classical model of stochastic frontier analysis (SFA), proposed simultaneously by Meeusen and van den Broek (1977) and Aigner et al. (1977) to estimate inefficiencies across individual firms, incorporates a composite error term which, under the null hypothesis of efficiency, is symmetric. We modify this framework to include additional dynamic effects that take into account potential serial correlation of Solow residuals and use it to estimate the ratio of variances of the symmetric and asymmetric error components.

Stochastic frontier analysis is an alternative to the VAR methodology used in the debate between Hamilton (2011), and Kilian and Vigfusson (2011a,b) over whether GDP responds symmetrically or asymmetrically to oil price innovations. In contrast to VAR analysis, SFA allows us to test for asymmetry without a hypothesis about the cause of the asymmetry. Therefore, our findings of asymmetry using SFA analysis are not related to the measurement of oil price variables or any other potentially causal variables. We make a separate attempt in the next section to model the cause of the asymmetries we find.

2.1 Model specification

We follow Barro (2006) and assume that the first difference of logged productivity is determined by a composite error. We use the data described below to construct a measure of the Solow residual \( SR_{Mt} \), and express the log-differenced Solow residual \( q_t \) as

\[
q_t = \log(SR_{Mt}) - \log(SR_{Mt-1}) = \mu_t + w_t,
\]

where, conditional on the information set generated by lagged Solow residuals, \( \mu_t \) is the conditional mean of \( q_t \), and \( w_t \) is a composite error term with mean zero. We assume that this error term can be decomposed as

\[
w_t = v_t - u_t,
\]

where \( v_t \) is a Gaussian random variable, \( v_t \sim N(\mu_u, \sigma^2_v) \) with mean \( \mu_u > 0 \). The second term \( u_t \) is an exponentially distributed (positive) random variable with mean \( \mu_u \). Note that by construction the mean of \( w_t \) is restricted to be zero. Other typical choices for the distribution of the one-sided error term \( u_t \) are half-normal, truncated normal, or gamma (Kumbhakar and Lovell 2000, Murillo-Zamorano 2004). For our data, described in Section 3, we found that the exponential distribution provides the best fit, so we restrict attention to this model. The presence of the term \( u_t \) implies that the distribution of the composite
error $w_t$ is asymmetric, except for the degenerate case in which $\mu_u = 0$ and the distribution reduces to a normal with variance $\sigma_v^2$.

The specification of the conditional mean $\mu_t$ depends on potential serial correlation of the growth rates of Solow residuals $q_t$. We consider autoregressive models of order $p$, AR($p$), given by

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + w_t,$$

where the usual stationarity conditions are assumed to be satisfied, and $w_t$ is the composite error term defined by equation (2).

### 2.2 Estimation and evaluation

Estimation of model (3), including the autoregressive coefficients and parameters in the distributions for $u_t$ and $v_t$ can be conveniently done by maximum likelihood.

A closed form expression for the density of the composite error $w_t = v_t - u_t$ exists and is given by (see Kumbhakar and Lovell 2000)

$$f(w_t; \mu_u, \sigma_v) = \frac{1}{\mu_u} \Phi \left( \frac{w_t - \mu_u}{\sigma_v} \right) \exp \left( \frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2} \right),$$

where $\Phi$ denotes the standard normal distribution function and $\mu_u$ is the parameter of the exponential distribution, which has mean equal to $\mu_u$ and variance equal to $\mu_u^2$. The mean of $u_t$ is subtracted from $w_t$ to account for the fact that, unlike in the classical stochastic frontier model, $w_t$ is assumed to have mean zero. The log-likelihood function of the joint model, including the autoregressive dynamics, can be obtained in a straightforward way.

A primary goal of this paper is to test whether the model, allowing for a one-sided error component in the AR innovations, outperforms the standard model with symmetric errors. The model with symmetric errors is nested in the model above when the parameter $\mu_u$ equals zero. Thus, one can test the null hypothesis that the benchmark model performs as well as our model by doing a likelihood ratio test. With the restriction $\mu_u \geq 0$, however, the parameter to test is on the boundary of the parameter space under the null hypothesis.

Lee (1993) showed that, in this case, the likelihood ratio statistic asymptotically follows a mixture of a $\chi^2$ distribution with one degree of freedom and a point mass of 1/2 at zero.

Estimation of this model is problematic when the sample skewness is positive. Aigner et al. (1977) demonstrated that theoretically in such situations the MLE of $\mu_u$ will converge to zero, and Lee (1993) showed that in this case the information matrix is singular.

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In our application we experimented with MA terms and found them unnecessary to capture dynamics.
which implies that maximum likelihood standard errors cannot be calculated. In practice, when the residuals have positive skewness, the MLE using (4) will either fail to converge or will converge to a local maximum. For cases in which the sample skewness of the residuals of the autoregressive model with Gaussian innovations is positive, we extend the model to explicitly allow for positive skewness. We allow \(-u_t\) to follow an exponential distribution, such that \(u_t\) has a negative mean \(\mu_u\) and standard deviation \(|\mu_u|\). This leads to the composite density of \(w_t\),

\[
f(w_t; \mu_u, \sigma_v) = \frac{1}{\mu_u} \Phi \left( -\frac{w_t - \mu_u}{\sigma_v} - \frac{\sigma_v}{\mu_u} \right) \exp \left( \frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_u^2}{2\mu_u^2} \right) I(\mu_u \geq 0) \\
- \frac{1}{\mu_u} \Phi \left( \frac{w_t - \mu_u}{\sigma_v} + \frac{\sigma_v}{\mu_u} \right) \exp \left( \frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_u^2}{2\mu_u^2} \right) I(\mu_u < 0),
\]

which for \(\mu_u > 0\) corresponds to the one given in (4) with negative skewness, and for \(\mu_u < 0\) is the mirrored version with positive skewness. \(I(\cdot)\) denotes the indicator function. Substituting (5) for equation (4) avoids convergence problems of the estimation algorithm due to positive sample skewness. Moreover, it is possible to use a standard \(\chi^2\) distribution for a likelihood ratio test because the parameter \(\mu_u\) is no longer on the boundary under the null hypothesis. Even if the estimate of \(\mu_u\) is negative, it could be insignificant and support the evidence of a symmetric error distribution. Note that this model specification is a vehicle to test our null hypothesis of interest, rather than our model of interest, which remains (4). We use equation (5) only when the estimation of equation (4) fails to converge due to positive skewness.

The relative importance of \(u_t\) can be measured by computing the variance ratio (VR) of the asymmetric component relative to the symmetric component as

\[
VR = \frac{\mu_u^2}{\sigma_v^2},
\]

which is estimated using the parameter estimates for the two error components. Note that the variance ratio measures the degree of asymmetry in the data with an increase in the ratio denoting more asymmetry. This is in contrast to the traditional stochastic frontier literature where the inefficiency of a firm is measured by technical efficiency (TE) defined by Battese and Coelli (1988) as \(E[\exp(-u)|w]\). TE measures the distance from the efficient frontier or actual output divided by optimal efficient output. However, in our context TE cannot be interpreted, because the expectation of \(u\) is absorbed by the mean of \(v\) and consequently only the variance of \(u\) relative to the variance of \(v\) is relevant.
3 Empirical Analysis of Asymmetries

The data set consists of seasonally-adjusted quarterly observations on constant-price GDP, investment, and employment for all OECD countries which had data dating back at least to the early 1980’s. Data is from Main Economic Indicators (MEI) from the OECD. The OECD does not have data on labor hours, a preferable measure for the labor input, or on capital stock. We include the following countries: Australia, Canada, France, Germany, Italy, Japan, Korea, Norway, Switzerland, United Kingdom and United States. The sample begins in 1973:Q1, with the exception of Korea (1983:Q1) and Switzerland (1976:Q1), and ends in 2011:Q3. Kilian and Vigfusson (2011b) argue that due to the regulation of nominal oil prices prior to 1973, the dynamic properties of oil prices in that period contain no information about the period following 1973. Therefore, our sample begins in 1973. The data set does not constitute a panel because units of measurement for each country’s output and investment differ since they are measured in country-specific units of 2005 GDP. Data from the Penn World Tables does adjust cross-country data to comparable units, using purchasing power parity measures of relative prices, but that data exists only at annual frequency. Since we are interested in business cycle properties, quarterly frequencies are essential. Therefore, we estimate eleven separate equations, decomposing the innovations of each country’s Solow residual into a symmetric and an asymmetric component.

To construct Solow residuals, we first construct measures of the capital stock. We use the perpetual inventory method, letting the initial value of the capital stock ($K_0$) be the steady-state equilibrium value with the growth rate ($g$) equal to the average of growth over the first ten years of the sample, annual depreciation ($\delta$) at 0.07, following Easterly and Levine (2001), and initial investment equal to its initial value ($I_0$). Subsequent values for capital are computed using the equation for the adjustment of the capital stock,

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (7)$$

To compute Solow residuals, we use employment as the measure of labor input and set capital’s share at 0.35, following Stock and Watson (1999).

Augmented Dickey-Fuller tests with and without a trend do not reject the hypothesis of a unit root in Solow residuals for all countries. Therefore, we compute the first

\[K_0 = \frac{I_0}{g + \delta}.\]

Since the sample begins with 1973Q1, $I_0$ is investment in this first period.
differences of the logarithm of the series. Before estimating our models we standard-
ize the difference of log Solow residuals to have mean zero and variance equal to one. 
This standardization does not affect the estimation of the quantities of interest such as 
the skewness or the variance ratio. Empirical autocorrelations and partial autocorre-
lations (not reported) suggest that low order autoregressive models capture the dynamics 
in the data and that no significant autocorrelation is present for many series. The lag 
length was chosen using a combination of information criteria and the Ljung-Box test 
for residual autocorrelation up to 24 lags. We allowed for the possibility of dropping 
insignificant intermediate autoregressive lags to reduce the number of parameters to be 
estimated. However, we checked the robustness of our results with respect to that choice. 
The estimated mean equations can be found in Table 5 in the appendix.

We present the estimation results from our baseline dynamic stochastic frontier (DSF) 
model, defined in equations (1) to (3), in Table 1. The table contains the estimated 
parameters of the composite error term, the sample skewness of the residuals \( w_t \), the log-
likelihood of both the model restricted to symmetry (LL sym) and our dynamic stochastic 
frontier model (LL DSF), the likelihood ratio statistic (LR stat) for the null of symmetry 
along with its p-value, and the variance ratio (VR) implied by the estimated model. 
When the residuals from the Gaussian model exhibit positive skewness, we estimate the 
extended model (5), allowing computation of standard errors and construction of the 
likelihood ratios. Note, however, that in these cases, rejection of the null hypothesis of 
symmetry provides evidence of positive asymmetry. Also recall that whenever we use 
the extended model (5) we use the \( \chi^2 \) distribution to compute p-values rather than the 
mixture distribution with point mass at 0 that is required when relying on (4).

The likelihood ratio test rejects the null hypothesis of symmetry against negative 
asymmetry for all countries except Australia, Canada and Norway. For Australia the 
symmetric model is rejected, but in favor of the model with positive skewness. Canada and 
Norway have no asymmetries, while the remaining countries all have significant negative 
asymmetries.

It is interesting to consider which countries fail to exhibit negative asymmetries. Both 
Canada and Norway are oil exporters. Australia is a net oil importer, but a net exporter 
of energy. The UK’s experience is mixed, with net exports from 1981 to 1988 and from 
1993 to 2003, and net imports in other periods (Bolton 2010). Therefore, the estimation 
provides evidence that all countries with some history of net energy imports have signif-
icient negative asymmetries. Countries which were consistently net exporters of energy

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do not. These results suggest that oil prices could have a role in creating asymmetries in Solow residuals for countries which are net importers of energy.
Table 1: Estimation results of the DSF model

<table>
<thead>
<tr>
<th></th>
<th>Aus</th>
<th>Can</th>
<th>Fra</th>
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<th>Nor</th>
<th>Swi</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_u$</td>
<td>-0.57***</td>
<td>-0.37</td>
<td>0.50***</td>
<td>0.57***</td>
<td>0.69***</td>
<td>0.56***</td>
<td>-0.56***</td>
<td>0.60***</td>
<td>0.64***</td>
<td>0.60***</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.80***</td>
<td>0.85***</td>
<td>0.82***</td>
<td>0.80***</td>
<td>0.69***</td>
<td>0.71***</td>
<td>0.78***</td>
<td>0.80***</td>
<td>0.61***</td>
<td>0.74***</td>
<td>0.79***</td>
</tr>
<tr>
<td>skewness</td>
<td>0.318</td>
<td>0.116</td>
<td>-0.161</td>
<td>-0.519</td>
<td>-0.635</td>
<td>-0.781</td>
<td>-0.689</td>
<td>0.269</td>
<td>-2.631</td>
<td>-0.391</td>
<td>-0.262</td>
</tr>
<tr>
<td>LL sym.</td>
<td>-214.4</td>
<td>-202.5</td>
<td>-211.5</td>
<td>-216.6</td>
<td>-203.6</td>
<td>-218.0</td>
<td>-161.3</td>
<td>-212.8</td>
<td>-191.9</td>
<td>-218.0</td>
<td>-218.0</td>
</tr>
<tr>
<td>LL DSF</td>
<td>-212.5</td>
<td>-202.3</td>
<td>-210.0</td>
<td>-213.0</td>
<td>-199.1</td>
<td>-211.6</td>
<td>-156.0</td>
<td>-211.8</td>
<td>-172.6</td>
<td>-212.7</td>
<td>-215.9</td>
</tr>
<tr>
<td>LR stat.</td>
<td>3.838</td>
<td>0.360</td>
<td>3.020</td>
<td>7.265</td>
<td>9.010</td>
<td>12.73</td>
<td>10.42</td>
<td>1.918</td>
<td>38.52</td>
<td>10.56</td>
<td>4.302</td>
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<tr>
<td>p-val.</td>
<td>0.050</td>
<td>0.549</td>
<td>0.041</td>
<td>0.004</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.166</td>
<td>0.000</td>
<td>0.001</td>
<td>0.019</td>
</tr>
<tr>
<td>VR</td>
<td>0.513</td>
<td>0.196</td>
<td>0.376</td>
<td>0.501</td>
<td>0.740</td>
<td>0.949</td>
<td>0.518</td>
<td>0.495</td>
<td>0.963</td>
<td>0.750</td>
<td>0.581</td>
</tr>
</tbody>
</table>

Note: Table 1 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter $\mu_u$ and a two-sided Gaussian r.v. with parameter $\sigma_v$. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.
4 Evidence on Determinants of Asymmetries

We have established that net oil importers have asymmetric innovations to TFP, as measured by Solow residuals. In this section, we consider alternative asymmetric variables that could account for the asymmetry. We consider two candidates, Hamilton’s net oil price increase variable and a country-specific financial crisis variable. We measure the oil price by deflating the spot price of oil by the US CPI to obtain the real oil price. We create the financial crisis dummy by using information on dates of financial crises for each country.

We assume that the conditional mean of our TFP variable depends on its own lags and on the lags of the additional explanatory variables, net oil price increase and financial crises dummies. Therefore, we augment the baseline model (3) by a vector of lagged, explanatory variables \( x \), including oil prices and financial crises, yielding

\[
q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + \sum_{i=1}^{p'} \alpha_i x_{t-i} + \tilde{w}_t,
\]

where \( \alpha \) is a vector of coefficients. This is equivalent to the first equation in a VAR in which TFP is ordered first. Equivalently, we are assuming that contemporaneous changes in either the net oil price or financial crisis takes at least one quarter to affect TFP. The equation is valid whether the variables in \( x_t \) are endogenous or exogenous to \( q_t \). That is, this equation is consistently estimated with OLS irrespective of the coefficients in the regression of \( x_t \) on \( q_t \) and lagged values of both. The conditional mean specifications for all countries are summarized in Table 5 in the appendix.

We reestimate the SFA model using the estimated value of \( \tilde{w}_t \) to determine whether negative asymmetries remain, after conditioning on Hamilton’s net oil price increase variable and on financial crises.

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5 We use the spot price of West Texas Intermediate from the FRED database for the nominal oil price. The US CPI is the consumer price index for all urban consumers from BLS. We use this real measure of oil price for the US and other countries. An alternative for other countries would have been to convert the dollar price of oil into local currency price, using the exchange rate, and then deflate by the relevant foreign consumer price index. We did not take this route because this method would identify exchange rate crises as periods of large oil price increases.

6 Therefore, the large literature on whether oil price changes are endogenous or exogenous is not relevant to our analysis.
4.1 Role of net oil price increase

First, we determine whether an asymmetric oil-price variable can reduce asymmetries. We use a measure of the real oil price, created by deflating the oil price, measured by the price of West Texas Intermediate (WTI), by the CPI, with both data taken from the FRED data base. Hamilton (2003, 2011) defines a variable that becomes effective only when the oil price attains a new 3-year high. With $X_t$ denoting the logarithm of the real oil price in period $t$, net oil price increase is defined by

$$oil_t = \max(0, X_t - \max(X_{t-1}, \ldots, X_{t-12})).$$

(9)

Recall that the UK was a net energy importer for some periods and a net exporter for others. Therefore, we define a dummy variable, identifying the periods in which it was a net importer (1973Q1-1980Q2, 1988Q3-1992Q4 and 2003Q3-2011Q3). The dummy enters both as an intercept dummy and as a multiplier on $oil_t$, to allow the effect of oil prices to differ across the two regimes.

The estimation results of the models containing oil price shocks, defined by equation (9), are shown in Table 2. Statistics for skewness of the residuals and the estimated variance ratio demonstrate that the asymmetry is reduced for all countries that initially showed negative asymmetry, with the exception of Korea. For a subset of these countries, France and Italy, the null hypothesis of symmetry can no longer be rejected. For the UK we now have some evidence for positive asymmetry. For the US, the p-value of the symmetry hypothesis increases from 0.019 to 0.081. The net oil price increase is a significant contributor to negative asymmetries in Solow residuals for net energy importers, and by implication, to asymmetries in business cycles for these countries. However, oil price shocks do not completely eliminate asymmetry for all countries. Even after conditioning on large oil price increases, Germany, Japan, Korea, and Switzerland continue to exhibit negative asymmetries. Hence, in the following we consider another potential source for asymmetry.

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7 The CPI code is CPIAUCSL.
8 We also tried alternative functions of oil prices to capture the nonlinear effects, and results were similar.
Table 2: Estimation results of the DSF model with oil

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<td>0.62***</td>
<td>0.56***</td>
<td>-0.53***</td>
<td>0.57***</td>
<td>-0.43***</td>
<td>0.51***</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.76***</td>
<td>0.84***</td>
<td>0.79***</td>
<td>0.79***</td>
<td>0.74***</td>
<td>0.70***</td>
<td>0.76***</td>
<td>0.81***</td>
<td>0.62***</td>
<td>0.80***</td>
<td>0.80***</td>
</tr>
<tr>
<td>skewness</td>
<td>0.352</td>
<td>0.108</td>
<td>0.220</td>
<td>-0.405</td>
<td>-0.139</td>
<td>-0.632</td>
<td>-0.747</td>
<td>0.241</td>
<td>-2.496</td>
<td>0.13</td>
<td>-0.105</td>
</tr>
<tr>
<td>LL sym.</td>
<td>-209.7</td>
<td>-199.7</td>
<td>-200.5</td>
<td>-209.0</td>
<td>-184.2</td>
<td>-207.1</td>
<td>-159.5</td>
<td>-209.8</td>
<td>-188.7</td>
<td>-203.9</td>
<td>-206.6</td>
</tr>
<tr>
<td>LL DSF</td>
<td>-207.6</td>
<td>-199.6</td>
<td>-199.1</td>
<td>-206.4</td>
<td>-183.8</td>
<td>-202.8</td>
<td>-154.0</td>
<td>-209.0</td>
<td>-172.0</td>
<td>-201.1</td>
<td>-205.6</td>
</tr>
<tr>
<td>LR stat.</td>
<td>4.178</td>
<td>0.268</td>
<td>2.754</td>
<td>5.074</td>
<td>0.734</td>
<td>8.601</td>
<td>11.01</td>
<td>1.577</td>
<td>33.32</td>
<td>5.572</td>
<td>1.957</td>
</tr>
<tr>
<td>p-val.</td>
<td>0.041</td>
<td>0.605</td>
<td>0.097</td>
<td>0.012</td>
<td>0.196</td>
<td>0.002</td>
<td>0.001</td>
<td>0.209</td>
<td>0.000</td>
<td>0.018</td>
<td>0.081</td>
</tr>
<tr>
<td>VR</td>
<td>0.559</td>
<td>0.175</td>
<td>0.308</td>
<td>0.468</td>
<td>0.226</td>
<td>0.788</td>
<td>0.546</td>
<td>0.439</td>
<td>0.855</td>
<td>0.288</td>
<td>0.407</td>
</tr>
</tbody>
</table>

**Note:** Table 2 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter $\mu_u$ and a two-sided Gaussian r.v. with parameter $\sigma_v$. The conditional mean equation includes the nonlinear term defined in (9). The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.
Another source of business cycle asymmetries could be extreme events such as the recent world-wide financial crisis. Barro (2006) included financial crises in his list of rare events. Therefore, we augment our model by conditioning on dummy variables representing financial crises. We date the recent financial crisis in the third quarter of 2008, the time of the Lehman Brothers collapse on September 15, 2008. Since this crisis actually hit different countries in different periods, we allowed for contemporaneous effects and lags based on the finding of a preliminary model selection. However, the only case where a contemporaneous effect is present is for the UK, for which the crisis is known to have started earlier due the collapse of Northern Rick in 2008Q1. For the Asian countries, Japan and Korea, we added a dummy for the Asian crisis in the fourth quarter of 1997. For Switzerland we added another dummy for the first quarter of 1991 to account for a banking crisis following a sharp decline in real estate prices. Estimation results for the model including financial crises are contained in Table 3.

Conditioning on financial crises reduces negative asymmetries for all countries with initial negative asymmetry. These asymmetries are completely eliminated for France, Germany, and Switzerland, and we are unable to reject the null of positive asymmetries for Korea. The financial crisis dummy is not significant for Norway and Canada, but these countries did not have negative asymmetries initially. In Italy, Japan, Korea, the UK, and the US, negative asymmetries, measured by variance ratios, decrease, but we continue to reject the null of symmetry in favor of negative asymmetry for these countries. This evidence suggests that the financial crises alone are not responsible for all negative asymmetries observed in the data.

Finally, we allow conditioning on both oil price shocks and financial crises. The results are presented in Table 4. Evidence of negative asymmetry in the residuals has disappeared for all countries. Three countries with initial negative asymmetry, France, Korea, and the UK, now have significant positive asymmetry. This result suggests that a combination of the net oil price increase and financial crises is responsible for the observed negative asymmetries in the Solow residual.

To summarize, for France, negative asymmetry can be completely eliminated with either the oil price variable or the financial crisis dummy. For Italy, the UK, and possibly for the US, depending on the desired critical value, the oil price variable completely eliminates evidence for negative asymmetry. For Germany, Switzerland, and Korea, the financial crisis dummy alone completely eliminates asymmetry. And, finally, for Japan
and possibly the US, both oil prices and financial crises are needed to completely eliminate asymmetry. These results also confirm Barro’s (2006) assumption that the asymmetric component of the Solow residual is determined by shocks which are large and infrequent – in our sample net oil price increases and financial crises.
Table 3: Estimation results of the DSF model with crisis dummies

<table>
<thead>
<tr>
<th></th>
<th>Aus</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
<th>Kor</th>
<th>Swi</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{u}$</td>
<td>-0.57***</td>
<td>-0.40***</td>
<td>-0.36</td>
<td>0.52***</td>
<td>0.49***</td>
<td>-0.59***</td>
<td>-0.24</td>
<td>0.55***</td>
<td>0.53***</td>
</tr>
<tr>
<td>$\sigma_{v}$</td>
<td>0.78***</td>
<td>0.84***</td>
<td>0.84***</td>
<td>0.68***</td>
<td>0.74***</td>
<td>0.56***</td>
<td>0.65***</td>
<td>0.80***</td>
<td>0.80***</td>
</tr>
<tr>
<td>skewness</td>
<td>0.325</td>
<td>0.009</td>
<td>0.071</td>
<td>-0.576</td>
<td>-0.404</td>
<td>0.868</td>
<td>0.111</td>
<td>-0.251</td>
<td>-0.120</td>
</tr>
<tr>
<td>LL sym.</td>
<td>-212.8</td>
<td>-205.1</td>
<td>-200.0</td>
<td>-192.2</td>
<td>-198.8</td>
<td>-139.4</td>
<td>-147.8</td>
<td>-206.7</td>
<td>-209.0</td>
</tr>
<tr>
<td>LL DSF</td>
<td>-210.8</td>
<td>-204.4</td>
<td>-199.8</td>
<td>-188.3</td>
<td>-196.8</td>
<td>-133.6</td>
<td>-147.7</td>
<td>-203.1</td>
<td>-207.9</td>
</tr>
<tr>
<td>LR stat.</td>
<td>4.077</td>
<td>1.482</td>
<td>0.358</td>
<td>7.942</td>
<td>4.026</td>
<td>11.51</td>
<td>0.171</td>
<td>7.278</td>
<td>2.272</td>
</tr>
<tr>
<td>p-val.</td>
<td>0.044</td>
<td>0.223</td>
<td>0.550</td>
<td>0.002</td>
<td>0.022</td>
<td>0.001</td>
<td>0.679</td>
<td>0.004</td>
<td>0.066</td>
</tr>
<tr>
<td>VR</td>
<td>0.531</td>
<td>0.223</td>
<td>0.185</td>
<td>0.583</td>
<td>0.440</td>
<td>1.104</td>
<td>0.142</td>
<td>0.480</td>
<td>0.432</td>
</tr>
</tbody>
</table>

Note: Table 3 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter $\mu_u$ and a two-sided Gaussian r.v. with parameter $\sigma_v$. The conditional mean equation includes dummies for financial crises as described in the text. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.
Table 4: Estimation results of the DSF model with oil and crisis dummies

<table>
<thead>
<tr>
<th></th>
<th>Aus</th>
<th>Fra</th>
<th>Ger</th>
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<th>Kor</th>
<th>Swi</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_u$</td>
<td>-0.57***</td>
<td>-0.48***</td>
<td>-0.34</td>
<td>0.27</td>
<td>0.31**</td>
<td>-0.56***</td>
<td>-0.30</td>
<td>-0.36***</td>
<td>0.45***</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.75***</td>
<td>0.72***</td>
<td>0.82***</td>
<td>0.74***</td>
<td>0.78***</td>
<td>0.57***</td>
<td>0.60***</td>
<td>0.69***</td>
<td>0.80***</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.366</td>
<td>0.436</td>
<td>0.038</td>
<td>-0.035</td>
<td>-0.121</td>
<td>0.836</td>
<td>0.243</td>
<td>0.129</td>
<td>-0.033</td>
</tr>
<tr>
<td>LL sym.</td>
<td>-207.9</td>
<td>-194.8</td>
<td>-196.3</td>
<td>-178.1</td>
<td>-189.0</td>
<td>-137.6</td>
<td>-142.9</td>
<td>-197.2</td>
<td>-202.9</td>
</tr>
<tr>
<td>LL DSF</td>
<td>-205.6</td>
<td>-192.5</td>
<td>-196.1</td>
<td>-178.0</td>
<td>-188.8</td>
<td>-132.4</td>
<td>-142.6</td>
<td>-195.5</td>
<td>-202.3</td>
</tr>
<tr>
<td>LR stat.</td>
<td>4.545</td>
<td>4.650</td>
<td>0.295</td>
<td>0.226</td>
<td>0.265</td>
<td>10.41</td>
<td>0.599</td>
<td>3.449</td>
<td>1.374</td>
</tr>
<tr>
<td>p-val.</td>
<td>0.033</td>
<td>0.031</td>
<td>0.587</td>
<td>0.317</td>
<td>0.303</td>
<td>0.001</td>
<td>0.439</td>
<td>0.063</td>
<td>0.121</td>
</tr>
<tr>
<td>VR</td>
<td>0.589</td>
<td>0.446</td>
<td>0.174</td>
<td>0.138</td>
<td>0.160</td>
<td>0.972</td>
<td>0.257</td>
<td>0.272</td>
<td>0.317</td>
</tr>
</tbody>
</table>

Note: Table 4 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter $\mu_u$ and a two-sided Gaussian r.v. with parameter $\sigma_v$. The conditional mean equation includes the nonlinear term defined in (9) and dummies for financial crises as defined in the text. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two-sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.
5 Theory: Asymmetric Component of Solow Residuals

Our results demonstrate that conditioning on net oil price increases and financial crises eliminates negative asymmetric innovations in Solow residuals for OECD countries. Does this imply that we should model TFP as having exogenous symmetric and asymmetric innovations, with the asymmetric innovation comprised of net oil price increases and financial crises? To address this question, we need economic theory. We first consider oil prices.

5.1 Oil prices and asymmetric Solow Residuals

Hamilton (1996, 2011) introduces asymmetry in output by considering effects on demand due to the price increase itself and due to increased uncertainty after the oil price has crossed a recent high or low. He argues that the price crossing either the high or low benchmark raises uncertainty thereby reducing demand. Since demand falls, either when price rises to cross a previous high or falls to cross a previous low, its response to oil price changes is asymmetric. Combining the direct effect on demand with the effect operating through uncertainty implies that an increase in the price of oil beyond a benchmark reduces demand on both counts. In contrast, a reduction in the price of oil has a direct positive effect on demand, but an indirect negative effect through uncertainty, yielding effects which Hamilton finds offsetting. Hamilton (1996, 2011) constructs a variable he calls a net oil price increase, defined as an increase in price relative to the high over the previous three years, to measure the asymmetric oil price movements to which he finds output responds.

We are interested in the potentially asymmetric effects of oil prices on TFP, and not on the effects on output itself. We measure TFP with Solow residuals. The Solow residual combines an exogenous technology component with an endogenous response of value added to a change in imported intermediate inputs. Oil prices do not directly affect technology. However, an increase in the world price of energy, created by an increase in the price of oil, could affect measured TFP through imported intermediate inputs.

We construct a simple model of a representative firm, operating in an open economy, to show that the Solow residual for the representative firm responds non-linearly and asymmetrically to an increase in the relative price of imported energy. Concavity in production yields non-linearity, where large price increases have greater absolute effects.
than large price decreases, while measurement in constant dollars can change the sign of the response, yielding additional asymmetry. The model illustrates that the Solow residual should respond asymmetrically to an increase in energy prices, providing a supply-side argument to support Hamilton’s asymmetric output response.

5.1.1 Model setup

We assume that all countries have some ability to produce their own energy, and that this ability varies across countries. The representative firm in each country chooses how much energy to produce based on the price of imported energy and its own production costs, and imports the remainder. We assume that the price of imported energy is exogenous to the representative firm.

Since imported energy’s share is empirically increasing in price, we follow Hassler et al. (2012) and specify a CES production function for output as

\[
Y_t = \left[ (1 - \gamma) \left[ A_t K_t^\alpha L_t^{1-\alpha} \right]^{\epsilon/\epsilon} + \gamma \left[ A_t^E E_t \right]^{\epsilon/\epsilon} \right]^{1/\epsilon},
\]

(10)

where \(\gamma\) is imported energy’s share in the event that \(\epsilon = 1\), \(E_t\) is imported energy, \(A_t^E\) is technology in energy, and \(\epsilon\) is the elasticity of substitution between the capital-labor composite and energy. We assume \(\epsilon < 1\) to be consistent with empirical evidence that imported energy’s share is increasing in price. The value for \(\gamma\) determines the curvature of production with respect to imported energy. A country which produces all of its own energy domestically will have \(\gamma = 0\). We think about \(\gamma\) as a structural decision made by the representative firm based on its own production technology for energy and the imported price. This decision cannot be changed in the short run. However, if the price of energy gets too high, even with the current value for \(\gamma\), the firm could be better off producing all of its own energy and could choose to do so. This puts an upper bound on the price for which the firm will import.

We assume that the energy price is low enough that the representative firm chooses to import. In this case, the firm chooses the quantity of imported energy input to maximize

\[
\left[ (1 - \gamma) \left[ A_t K_t^\alpha L_t^{1-\alpha} \right]^{\epsilon/\epsilon} + \gamma \left[ A_t^E E_t \right]^{\epsilon/\epsilon} \right]^{1/\epsilon} - p_t E_t,
\]

where \(p_t\) is the relative price of energy in terms of production. First order conditions require that the marginal product of imported energy in production equal its relative price

\[
\frac{\partial Y_t}{\partial E_t} = p_t,
\]
yielding an expression for optimal imported energy as

$$E_t = Y_t p_t^{-\epsilon} \gamma^{\epsilon} \left( A_t^E \right)^{\epsilon - 1}.$$  \hspace{1cm} (11)

Using equation (11), imported energy’s share can be expressed as

$$\frac{p_t E_t}{Y_t} = \gamma \left( \frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon}.$$  \hspace{1cm} (12)

Therefore, with $\epsilon < 1$, imported energy’s share of production depends positively on its relative price. In the case with $\epsilon = 1$, imported energy’s share is simply $\gamma$, independent of price.

Substituting equation (11) for $E_t$ into equation (10) for production yields an expression for production as a function of the relative price of energy,

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} (1 - \gamma)^{\frac{1}{\epsilon}} \left[ 1 - \gamma \left( \frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$  \hspace{1cm} (13)

Using equations (11) and (13), we can solve for imported energy demand as a function of price and the capital-labor composite as

$$E_t = (1 - \gamma)^{\frac{1}{\epsilon}} \frac{A_t}{A_t^E} K_t^\alpha L_t^{1-\alpha} \left[ \left( \frac{p_t}{\gamma A_t^E} \right)^{\epsilon - 1} - \gamma \right]^{\frac{1}{\epsilon}}.$$  \hspace{1cm} (14)

The firm will choose to import positive quantities of energy only if the term in brackets is positive, implying an upper bound on the energy price for which it chooses to import. For $\epsilon < 1$, derivatives of equations (13) and (14) illustrate that an increase in the price of energy reduces energy imports, thereby reducing production for a given value for capital and labor.

Kehoe and Ruhl (2008) explain that we must calculate the Solow Residual from value added, not from production. Value added, not total production, is the GDP concept. Value added (GDP) is given by production minus the value of imports in terms of production. Using equations (12) and (13) yields

$$GDP_t = Y_t - p_t E_t = Y_t \left( 1 - \gamma \left( \frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right) = A_t K_t^\alpha L_t^{1-\alpha} (1 - \gamma)^{\frac{1}{\epsilon}} \left[ 1 - \gamma \left( \frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$
5.1.2 Solow residual in terms of a numeraire

The Solow Residual is calculated as value added divided by the capital-labor input, yielding

$$SR_t = \frac{Y_t - p_t E_t}{K_t^\alpha L_t^{1-\alpha}} = A_t \left( 1 - \gamma \right) \left( \frac{p_t}{\gamma A_t^E} \right)^{\frac{\gamma}{1-\gamma}}\left[ 1 - \gamma \left( \frac{p_t}{\gamma A_t^E} \right)^{\frac{\gamma}{1-\gamma}} \right].$$

(15)

Taking the derivative of the Solow Residual in equation (15) with respect to the energy price yields

$$\frac{\partial SR}{\partial p_t} = - (1 - \gamma) \frac{A_t}{A_t^E} \left[ \left( \frac{p_t}{\gamma A_t^E} \right)^{\frac{\gamma}{1-\gamma}} - \gamma \right] = \frac{-E_t}{K_t^\alpha L_t^{1-\alpha}},$$

where the last equality uses equation (14) for optimal imported energy. Therefore, an increase in the price of imported energy reduces imports, thereby reducing the Solow residual, reducing output for given capital and labor. The second derivative of the Solow residual with respect to price is positive for \( \epsilon < 1 \), implying that the Solow residual is falling at an increasing rate in price. These two derivatives imply a non-linear response of the Solow residual to energy price changes. The non-linearity creates an asymmetric response with positive price changes having a bigger absolute impact on the Solow residual than large negative changes. The asymmetry is more pronounced the larger the absolute value of the price change.

We can understand the effect of energy price increases and decreases using a diagram. Figure 1 graphs production, equation (10), as a function of imported energy for a given value of the capital labor composite \((K_t^\alpha L_t^{1-\alpha})\) as \(Y\), and the cost of imported energy for a given price, \(P_j\), as \(P_j E\). Under the assumption that energy price is low enough for the firm to prefer imports, the profit maximizing use of energy occurs at \(E_0\) where the slope of the production function equals the price of energy, equivalently, where the slope of \(Y\) equals the slope of \(P_0 E\). The value for \(GDP_0\) is given by the vertical distance between \(Y\) and \(P_0 E\) at the point where the slopes are equal. The Solow residual is the value of \(GDP_0\) divided by the capital-labor composite \((K_t^\alpha L_t^{1-\alpha})\).

Now, consider the effect of an increase in the price of imported energy \((P_j)\) on GDP, holding the capital-labor composite constant, under the assumption that the price increase is small enough that the representative firm continues to import energy. As price increases to \(P_1 > P_0\), the slope of \(P_j E\) rises, and equilibrium occurs where the slope of the production function is higher to match the higher energy price, at \(E_1\). Therefore, holding the capital-labor composite constant, energy use falls and GDP falls from \(GDP_0\) to
Since $GDP$ is lower for a given value for $K^\alpha L^{1-\alpha}$, the Solow residual is lower. A decrease in price would reduce the slope of the $P_j E$ line, increasing GDP, but by a smaller amount than the same size price increase raises GDP, due to concavity of the production function. Concavity in production yields a non-linear and asymmetric response of the Solow residual to price changes.

### 5.1.3 Solow residual in constant dollars

Although optimal firm decisions are based on measurement in terms of the relative price of imported energy in terms of production, Kehoe and Ruhl (2008) explain that this measure of the Solow Residual is not the measure in the national income accounts. The measure of real value added in the data is not in terms of a numeraire (production in our example) but in terms of constant dollars. Letting $\bar{p}$ denote the dollar price of imported
energy in the base year with the dollar price of output at unity, and using equation (11), we can express value added in constant dollars, which we call measured GDP ($GDP_M$) as

$$GDP_M^t = Y^t - \bar{p}E^t = Y^t \left[ 1 - \bar{p} p^{\epsilon - 1} \left( \frac{A^E_t}{\gamma A^E_t} \right)^{1 - \epsilon} \right] = Y^t \left[ 1 - \gamma \left( \frac{p^t}{\gamma A^E_t} \right)^{1 - \epsilon} \frac{\bar{p}}{p^t} \right]. \quad (16)$$

Using the first equality in equation (16), the derivative of value added with respect to $p^t$ is given by

$$\left( \frac{\partial Y^t}{\partial E^t} - \bar{p} \right) \frac{\partial E^t}{\partial p^t} = (p^t - \bar{p}) \frac{\partial E^t}{\partial p^t}.$$

Given that a firm optimally equates the marginal product of energy with the price, there is no effect of a marginal increase in the price of energy on measured value added if the price equals the baseline price of $\bar{p}$. However, when price exceeds its baseline value an increase in energy price takes on the negative sign of the effect of an increase in price on energy imports. And when the price is below the baseline, the effect of an increase in energy price on measured GDP is positive.

The effect of energy prices on the Solow residual mirrors their effect on value added. Using equations (13) and (16), the measured Solow Residual in the data can be expressed as

$$SR_M^t = GDP_M^t = A_t \left( 1 - \gamma \right)^{1 - \epsilon} \left[ 1 - \gamma \left( \frac{p^t}{\gamma A^E_t} \right)^{1 - \epsilon} \frac{\bar{p}}{p^t} \right],$$

and its derivative with respect to price is

$$\frac{\partial SR_M^t}{\partial p^t} = -\frac{p^t - \bar{p}}{p^t} (1 - \gamma)^{1 - \epsilon} \frac{A_t}{\gamma A^E_t} \left[ 1 - \gamma \left( \frac{p^t}{\gamma A^E_t} \right)^{1 - \epsilon} \right]^{-1} \left( 1 - \gamma \left( \frac{p^t}{\gamma A^E_t} \right)^{1 - \epsilon} \frac{\bar{p}}{p^t} \right). \quad (17)$$

The derivative of the measured Solow residual is proportional to the derivative of the Solow residual in terms of a numeraire when price is above the baseline; however, with price below the baseline, the sign is reversed.

To facilitate intuition, we illustrate the behavior of the measured Solow residual in constant dollars using Figure 1. After the increase in price, the equilibrium quantity of imported energy is allowed to change, but since price is in constant dollars, the price is fixed. Therefore, the measured value for GDP after the price increase equals production at the new optimal energy input of $E_1$ less energy inputs valued at $P_0$, not at $P_1$, yielding a
value for measured GDP as \( \text{GDP}^{1M} \). GDP falls only by the vertical distance \( d \). Therefore, the effect of the increase in the price of energy on GDP and on the Solow residual is much smaller in the data, and would be zero if there were no concavity in production with respect to imported energy. Indeed, an infinitesimally small price change, as with a derivative, implies no change in measured GDP.

An increase in the price of energy has very different effects on the Solow residual in terms of a numeraire, presented at the beginning of this section, and the measured Solow residual in constant dollars. The difference between the two measures lies in the way the cost of imported intermediate inputs are treated. When we compute “real” using a numeraire, the increase in energy price raises the cost of imported intermediate inputs, deteriorating the terms of trade and reducing GDP. When we use constant dollars, the increase in energy price does not count toward an increase in the cost of intermediate
inputs since this cost must be measured in constant dollars.

Now compare the effect of a price increase and a price decrease on measured Solow residuals using Figure 2. We assume that the quantity of imported energy is initially optimal at $E_0$. As price changes, $E$ moves in the opposite direction. GDP is the difference between the new value of production and the new value of energy imports measured at the initial unchanged price. Therefore, a price decrease raises imported energy to $E_1$ and decreases GDP by $d_1$, whereas a price increase reduces imported energy to $E_2$ and reduces GDP by $d_2$. Consequently, both positive and negative changes in price reduce GDP for a given capital-labor composite, thereby reducing the measure of the Solow residual in the data. Measurement in constant dollars creates an extreme asymmetric response of the Solow residual to energy price changes at the baseline price, with any non-infinitesimal price change having a negative effect.

The measured Solow residual is a hump-shaped function of the price as illustrated in Figure 3. The peak of the hump occurs at the baseline price of $\bar{p}$ where an infinitesimally small change in the price has no effect on the Solow residual, and a larger change reduces the Solow residual irrespective of the sign of the change. When price is above the baseline, say at $p_1$, placing the system at point A along the curve, then an increase in price moves the economy further from the optimal, reducing the value for the Solow residual. Due to concavity in production, the response to a given-sized price increase is larger the greater the initial distance from the baseline, with the response from $p_1$ at point A exceeding that from $p_0$ at point B. Additionally, concavity implies that the response is increasing at an increasing rate in the magnitude of the price increase and that when the price is above the baseline, the response to a price increase exceeds that for a price decrease. And due to the effect of constant dollars in creating the hump-shaped response, a reduction in price from $p_0$ to $p_2$ moves the economy from point B to point C, reducing the Solow residual from $SR_0$ to $SR_2$.

In general, the hump-shaped response implies an asymmetric response of the Solow residual to oil price changes. If price is above the baseline, then a price increase reduces the Solow residual whereas a price decrease will have a smaller absolute effect and could even reduce the Solow residual. The effects are reversed when price is below the baseline, with a price decrease having the predictable effect of reducing the Solow residual and a price increase having a smaller effect, possibly with changed sign. The asymmetries depend on both the value of the initial price compared to the baseline and on the magnitude of the price change.
Consider how Hamilton’s net oil price increase variable captures the asymmetries in Figure 3. The base year in our sample is 2005. There are 33 non-zero observations on net oil price increase and 30 of the 33 observations non-zero observations occur with the initial price in the Hamilton variable, which we denote as $p_{t-i}$, greater than the baseline price of $\bar{p}$. Figure 3 can be used to show that any time the initial price exceeds the baseline and rises, the Solow residual must fall. Given that 30 of the 33 values for net oil price increase occur with $p_{t-i} > \bar{p}$, our regression of the change in the Solow residual on the net oil price increase is capturing the negative relationship given by the downward-sloping portion of the curve. Were the variable to include all oil price increases, many increases would occur from positions for which $p_{t} < \bar{p}$, a portion of the curve along which the slope is opposite. The variable also omits price decreases, excluding observations involving a switch between the downward and upward sloping portions of the curve.

The non-linearity in the measured Solow residual is extreme and cannot be entirely captured with a single variable. We can provide a measure of the estimate of the slope
of a portion of the curve only if we restrict the range. Hamilton’s net oil price increase variable restricts the range, thereby estimating the slope along the downward-sloping portion of the curve. However, since it is a single variable, it does not provide estimates of the non-linearities implied by the model over this range. Conditioning on the net oil price increase variable does reduce negative asymmetries in the innovation to the Solow residual for most countries, with the implication that other oil price changes have either symmetric or negligible effects.

Additionally, we can use the model to argue that the restriction provides an estimate of the slope of the most relevant portion of the curve. From equation (17), the relationship between the Solow residual and oil prices along the downward-sloping portion of the curve is proportional to the relationship present when the Solow residual is measured in terms a numeraire instead of in constant dollars. Firm decisions, including equilibrium employment, respond to the Solow residual measured in terms of a numeraire, not the constant-dollar measure in the data. Therefore, with employment responding to the actual Solow residual instead of the measured one, our combined theoretical and empirical results are consistent with Hamilton’s (1983) original hypothesis, that large oil price increases are associated with recessions which are much larger than the booms associated with large oil price reductions.

5.1.4 Discussion

These results imply that changes in oil prices should have asymmetric and non-linear effects on the measured Solow residual for countries with net energy imports. There should be no negative asymmetries for countries which are net exporters of energy. Return to the question of whether we can consider the net oil price increase to be an asymmetric exogenous shock to measured TFP. The model of the representative firm takes the change in the oil price to be exogenous. Therefore, we initially consider the case of an increase in the oil price which is independent of the country’s own technology, a characterization of small economies generally and of large economies in some cases.

Our results imply that measured TFP does have an asymmetric innovation measured by the net oil price increase. However, our analysis of the response to oil prices implies that these asymmetric negative TFP innovations are not true technology shocks, but rather the endogenous responses of imported intermediate inputs to a world shock, where the combined response is asymmetric and non-linear. A properly specified theoretical model must allow for the asymmetry, but not by adding an asymmetric exogenous innovation to
TFP because the innovation is endogenous.

For large countries, there is an additional possibility. An exogenous increase in the technology component of TFP itself for a large country could cause a net oil price increase by raising world demand for energy. The representative firm in the large economy takes world energy prices as exogenous and therefore responds asymmetrically to large energy price increases, compared with large decreases. Whether the net oil price increase is endogenous or exogenous to the large country’s exogenous technology shocks, the asymmetric response of TFP is the endogenous response of value-added in constant dollars.

Therefore, our model argues for allowing for the endogenous and asymmetric response of measured TFP, through changes in imported intermediate inputs, to shocks which ultimately cause oil price changes. The asymmetric response to price increases and price decreases is due to the combined effect of concavity in production and measurement in constant dollars. With only concavity in production, effects would be non-linear with price increases having larger effects than price decreases, but the sign of the effect of a price increase would never reverse. There is no case for adding an exogenous asymmetric TFP shock, measured by net oil price increases. Measured TFP does have an asymmetric component due to oil prices, but this component is an endogenous response to changes in oil prices.

5.2 Financial crises and asymmetric Solow residuals

In our framework a financial crisis is an extreme negative event, as in Barro (2006), and as such is explicitly an asymmetric shock. However, a financial crisis is not technology, and a financial crisis does not explicitly destroy technology. What is the mechanism by which a financial crisis creates an asymmetric innovation in TFP?

A model of financial crises is beyond the scope of this paper. However, we can get effects of a financial crisis on TFP if we modify the model to allow endogenous capacity utilization. It seems plausible that the disruption of finance together with its increased costs could reduce the firm’s capacity utilization on the supply side, and the recession and the accompanying fall in demand could operate on the demand side. Therefore, the asymmetry due to financial crises is likely to be the endogenous response of capacity utilization to the asymmetric financial crisis shock, not an exogenous asymmetric TFP shock. Financial crises could create asymmetry in measured TFP through the endogenous response of capacity to the asymmetric shock.
6 Conclusion

We investigate business cycle asymmetry by estimating the degree of asymmetry present in total factor productivity, as measured by Solow residuals, for eleven OECD countries. Barro (2006) introduced the idea that total factor productivity could have two components, one which is distributed normal i.i.d., as in DSGE models, and one which has only negative realizations. Barro identified his asymmetric component with events which reduced real GDP by at least fifteen percent. In contrast to Barro, we estimate the extent of asymmetry present in Solow residuals using Stochastic Frontier Analysis.

We perform likelihood ratio tests to determine whether the Solow residual contains a negative asymmetric component and compute variance ratios to estimate the extent of the asymmetry. We find that eight of the eleven OECD countries in our sample have significant negative asymmetries. Additionally, we use the pattern of asymmetry across countries to understand its cause. All countries with significant negative asymmetries are net energy importers and all countries without are net energy exporters. This pattern leads us to consider whether oil prices can explain some of the asymmetry.

We condition Solow residuals on Hamilton’s net oil price increase variable and determine whether asymmetries are reduced. We find that asymmetries are reduced for all countries which initially had them, except for Korea, and that for some countries, there is no longer evidence of negative asymmetries. This provides additional support for the hypothesis that oil prices bear some responsibility for business cycle asymmetry, but some asymmetry does remain for some countries. Finally, we introduce another cause of asymmetry, following Barro (2006), and condition on financial crises. Conditioning on financial crises alone reduces but does not eliminate asymmetry. However, conditioning on both the net oil price increase variable and financial crises completely eliminates any evidence of negative asymmetry in the Solow residuals.

To summarize, a model which allows negative asymmetries in the Solow residual is a better fit for eight of eleven OECD countries than a model which requires symmetry. The countries with negative asymmetries all have a history of net oil imports. Conditioning on net oil price increases reduces or eliminates the asymmetries for all countries except Korea. Complete elimination of negative asymmetries for all countries requires the addition of financial crisis dummies. Our results empirically substantiate Barro’s (2006) hypothesis that negative asymmetries in the Solow residual are due to extreme events, and add large net oil price increases, proposed by Hamilton (2003, 2011), to Barro’s list of extreme events.
Finally, we introduce a theoretical model to consider whether our results require the addition of an asymmetric TFP innovation to the standard symmetric innovations. Although conditioning on asymmetric variables eliminates negative asymmetries, we argue that the asymmetries are endogenous responses to oil and financial crisis shocks and are not exogenous shocks themselves. Therefore, models must be set up to allow asymmetric responses, but not to have asymmetric exogenous TFP innovations. Extreme events, including oil price changes and financial crises, create endogenous asymmetry in measured TFP.
# A Conditional mean equations

<table>
<thead>
<tr>
<th>Country</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aus</td>
<td>$q_t = 0.0018 - 0.1361q_{t-1} + w_t$</td>
</tr>
<tr>
<td>Aus-Oil</td>
<td>$q_t = 0.0637 - 0.1631q_{t-1} - 2.7323oil_{t-1} + w_t$</td>
</tr>
<tr>
<td>Aus-Crisis</td>
<td>$q_t = 0.0121 - 0.141q_{t-1} - 1.5815cri_{t-1} + w_t$</td>
</tr>
<tr>
<td>Aus-Oil + Crisis</td>
<td>$q_t = 0.0755 - 0.1687q_{t-1} - 2.7768oil_{t-1} - 1.6501cri_{t-1} + w_t$</td>
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<td>Can</td>
<td>$q_t = -0.0017 + 0.2467q_{t-1} + 0.277q_{t-3} + w_t$</td>
</tr>
<tr>
<td>Can-Oil</td>
<td>$q_t = 0.0467 + 0.2228q_{t-1} + 0.2825q_{t-3} - 2.1028oil_{t-2} + w_t$</td>
</tr>
<tr>
<td>Fra</td>
<td>$q_t = -0.0040 + 0.2597q_{t-1} + w_t$</td>
</tr>
<tr>
<td>Fra-Oil</td>
<td>$q_t = 0.0917 + 0.2630q_{t-1} - 4.1991oil_{t-3} + w_t$</td>
</tr>
<tr>
<td>Fra-Crisis</td>
<td>$q_t = 0.0164 + 0.2214q_{t-1} - 3.2105cri_{t-1} + w_t$</td>
</tr>
<tr>
<td>Fra-Oil + Crisis</td>
<td>$q_t = 0.1080 + 0.2178q_{t-1} - 4.0676oil_{t-3} - 2.9506cri_{t-1} + w_t$</td>
</tr>
<tr>
<td>Ger</td>
<td>$q_t = w_t$</td>
</tr>
<tr>
<td>Ger-Oil</td>
<td>$q_t = 0.0668 - 2.9001oil_{t-3} + w_t$</td>
</tr>
<tr>
<td>Ger-Crisis</td>
<td>$q_t = 0.0488 - 2.7915cri_{t-1} - 4.4188cri_{t-2} + w_t$</td>
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<tr>
<td>Ger-Oil + Crisis</td>
<td>$q_t = 0.1019 - 2.480oil_{t-3} - 2.686cri_{t-1} - 3.941cri_{t-2} + w_t$</td>
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<tr>
<td>Ita</td>
<td>$q_t = -0.0175 + 0.3047q_{t-1} + w_t$</td>
</tr>
<tr>
<td>Ita-Oil</td>
<td>$q_t = 0.1108 + 0.1917q_{t-1} - 4.1726oil_{t-3} - 2.2682oil_{t-4} + w_t$</td>
</tr>
<tr>
<td>Ita-Crisis</td>
<td>$q_t = -0.0011 + 0.2163q_{t-1} - 2.0747cri_{t-1} - 3.2797cri_{t-2} + w_t$</td>
</tr>
<tr>
<td>Ita-Oil + Crisis</td>
<td>$q_t = 0.1266 + 0.1501q_{t-1} - 3.792oil_{t-3} - 2.2031oil_{t-4} - 1.5714cri_{t-1} - 2.5118cri_{t-2} + w_t$</td>
</tr>
<tr>
<td>Jap</td>
<td>$q_t = w_t$</td>
</tr>
<tr>
<td>Jap-Oil</td>
<td>$q_t = 0.1244 - 3.5755oil_{t-3} - 1.9212oil_{t-3} + w_t$</td>
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<tr>
<td>Jap-Crisis</td>
<td>$q_t = 0.0634 - 3.4232cri_{t-1} - 3.8860cri_{t-2} - 2.4627asi_{t} + w_t$</td>
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<tr>
<td>Jap-Oil + Crisis</td>
<td>$q_t = 0.1787 - 3.5868oil_{t-3} - 1.5569oil_{t-3} - 3.3728cri_{t-1} - 3.6023cri_{t-2} - 2.5119asi_{t} + w_t$</td>
</tr>
<tr>
<td>Kor</td>
<td>$q_t = w_t$</td>
</tr>
<tr>
<td>Kor-Oil</td>
<td>$q_t = 0.0630 - 4.0260oil_{t-3} + w_t$</td>
</tr>
<tr>
<td>Kor-Crisis</td>
<td>$q_t = 0.0717 - 3.8978cri_{t-1} - 4.2755asi_{t} + w_t$</td>
</tr>
<tr>
<td>Kor-Oil + Crisis</td>
<td>$q_t = 0.1101 - 2.4806oil_{t-3} - 3.7956cri_{t-1} - 4.3322asi_{t} + w_t$</td>
</tr>
<tr>
<td>Nor</td>
<td>$q_t = -0.0023 + 0.2506q_{t-1} + w_t$</td>
</tr>
<tr>
<td>Nor-Oil</td>
<td>$q_t = 0.0369 - 0.2497q_{t-1} - 1.8121oil_{t-3} + w_t$</td>
</tr>
</tbody>
</table>

Table 5: Conditional mean equations
Table 5 – continued

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swi</td>
<td>[ q_t = -0.0074 + 0.1840q_{t-1} + w_t ]</td>
</tr>
<tr>
<td>Swi-Oil</td>
<td>[ q_t = -0.0424 + 0.1666q_{t-1} + 1.9006oil_{t-1} + w_t ]</td>
</tr>
<tr>
<td>Swi-Crisis</td>
<td>[ q_t = 0.0329 + 0.3775q_{t-1} - 2.3414cri_{t-1} - 6.676swi\cdot cri_{t-2} + w_t ]</td>
</tr>
<tr>
<td>Swi-Oil + Crisis</td>
<td>[ q_t = 0.0563 + 0.3640q_{t-1} + 2.6688oil_{t-2} - 4.0414oil_{t-3} - 2.6698cri_{t-1} - 6.6495swi\cdot cri_{t-2} + 3.7708swi\cdot cri_{t-3} + w_t ]</td>
</tr>
<tr>
<td>Uk</td>
<td>[ q_t = w_t ]</td>
</tr>
<tr>
<td>Uk-Oil</td>
<td>[ q_t = 0.3382 - 0.5059d_{import} - 3.3368oil_{t-3}d_{import_{t-3}} + w_t ]</td>
</tr>
<tr>
<td>Uk-Crisis</td>
<td>[ q_t = 0.0626 - 2.5456cri_{t} - 3.1156cri_{t-1} + w_t ]</td>
</tr>
<tr>
<td>Uk-Oil + Crisis</td>
<td>[ q_t = 0.3080 - 0.3574d_{import} - 2.9683oil_{t-3}d_{import_{t-3}} - 1.8867cri_{t} - 2.7954cri_{t-1} + w_t ]</td>
</tr>
<tr>
<td>US</td>
<td>[ q_t = w_t ]</td>
</tr>
<tr>
<td>US-Oil</td>
<td>[ q_t = 0.1268 - 3.1609oil_{t-2} - 1.6855oil_{t-3} + w_t ]</td>
</tr>
<tr>
<td>US-Crisis</td>
<td>[ q_t = 0.0327 - 3.1611cri_{t-1} + w_t ]</td>
</tr>
<tr>
<td>US-Oil + Crisis</td>
<td>[ q_t = 0.1321 - 2.6999oil_{t-2} - 1.6587oil_{t-3} - 2.5744cri_{t-1} + w_t ]</td>
</tr>
</tbody>
</table>

Note: Table 5 reports the estimated conditional mean equations for our dynamic stochastic frontier model defined by equations (2) to (5). The variable \( oil_t \) is defined in equation (9). The crisis dummies \( cri_{t} \), \( asi\cdot cri_{t} \), and \( swi\cdot cri_{t} \) represent 2008Q3, 1997Q4 and 1991Q1, respectively. The dummy \( d_{import_t} \) takes on the value one for periods when the UK was a net energy importer, namely 1973Q1-1980Q2, 1988Q3-1992Q4 and 2003Q3-2011Q3. All parameter estimates with the exception of the intercepts are significant at least at the 10% level of significance.
References


